

# Monopoles, Vortices and Confinement in $SU(3)$ Lattice Gauge Theory

Roy Wensley <sup>a</sup> and John Stack <sup>b</sup> \*

<sup>a</sup>Department of Physics and Astronomy, Saint Mary's College, Moraga, California, 94575, USA

<sup>b</sup>Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois, 68101

We present results for the heavy quark potential computed in  $SU(3)$  from magnetic monopoles and from center vortices. The monopoles are identified after fixing  $SU(3)$  lattice configurations to the maximal abelian gauge. The center vortices are identified after using an indirect center gauge fixing scheme which we describe for  $SU(3)$ .  $Z(3)$  center vortices are extracted and used to compute the potential. The values of the string tensions from monopoles and vortices are compared to the full  $SU(3)$  string tension.

A clear understanding of the confinement mechanism in QCD is still elusive. Two particular mechanisms have gained the most attention: magnetic monopoles and center vortices. Here we present results of calculations in  $SU(3)$  pure gauge theory of the heavy quark potential using methods which test these two different mechanisms.

## 1. Monopoles and the Maximal Abelian Gauge

The maximal abelian gauge is obtained in  $SU(3)$  gauge theory by maximizing the lattice functional

$$S = \sum_{x,\mu} \frac{1}{2} [Tr(U_\mu(x)\lambda_3 U_\mu^\dagger(x)\lambda_3) + Tr(U_\mu(x)\lambda_8 U_\mu^\dagger(x)\lambda_8)],$$

where  $\lambda_8$  and  $\lambda_3$  are the diagonal generators of the fundamental representation of  $SU(3)$ . The maximization is done by performing local  $SU(3)$  gauge transformations on the links, which in practice is done using an  $SU(2)$  subgroup update[1].

Once the maximal abelian gauge is obtained, two independent abelian gauge fields are projected out, leaving  $U(1) \times U(1)$  gauge fields. Magnetic monopoles are then identified in the abelian

configurations. The lattice monopoles are used to directly compute the heavy quark potential[2–4].

## 2. Indirect Center Gauge

The indirect center gauge was used in the first lattice studies of projected center vortices in  $SU(2)$ [5]. The indirect center gauge is obtained in the following way. A non-abelian lattice gauge configuration is fixed to the maximal abelian gauge. Abelian projection is performed obtaining the abelian subgroup configurations: In  $SU(2)$  this procedure obtains  $U(1)$  gauge fields, while in  $SU(N)$  it yields  $[U(1)]^{N-1}$  gauge fields. The remaining  $U(1)$  configurations are then gauge fixed so that the gauge fields are brought as close as possible to the elements of the center group of the original  $SU(N)$  theory. For the case of  $SU(2)$  this corresponds to maximizing

$$R_2 = \sum_{x,\mu} \cos^2(\theta(x,\mu)),$$

where  $\theta(x,\mu)$  parameterizes the abelian links. The maximization is obtained by iteratively performing  $U(1)$  gauge transformations. The final step to extract the vortices involves another projection of the lattice gauge fields onto the group elements of  $Z(2) \in \{-1, +1\} \times \mathbf{1}$ , where  $\mathbf{1}$  is the unit matrix.

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In  $SU(3)$  after maximal abelian gauge fixing and projection is done, the links are elements of  $U(1) \times U(1)$ . The  $U(1) \times U(1)$  links can be parameterized by the matrix:

$$U_\mu(x) = \text{diag}(e^{i\theta_1(x,\mu)}, e^{i\theta_2(x,\mu)}, e^{-i(\theta_2(x,\mu)+\theta_1(x,\mu))}).$$

$U(1) \times U(1)$  gauge transformations are then applied to these links to bring them as close as possible to the elements of the  $SU(3)$  center group,

$$Z(3) \in \{\exp(-i2\pi/3), 1, \exp(+i2\pi/3)\} \times \mathbf{1}.$$

The necessary gauge transformations can be parameterized using the fundamental, diagonal generators of  $SU(3)$ :

$$g(x) = e^{i\tilde{\lambda}_8\alpha_2(x)} e^{i\lambda_3\alpha_1(x)},$$

where some numerical factors are absorbed into the  $\alpha$ 's and we use

$$\tilde{\lambda}_8 = \text{diag}(1, 1, -2)$$

$$\lambda_3 = \text{diag}(1, -1, 0).$$

The center gauge in  $SU(N)$  is obtained by maximizing the quantity

$$R_N = \sum_{x,\mu} |\text{Tr} U_\mu(x)|^2.$$

For the indirect case in  $SU(3)$ , this is accomplished by applying the  $U(1) \times U(1)$  gauge transformations described above to the diagonal  $U(1) \times U(1)$  links to maximize the quantity

$$R_3 = \sum_{x,\mu} 3 + 2 \cos(\theta_1(x, \mu) - \theta_2(x, \mu)) \\ + \cos(\theta_1(x, \mu) + 2\theta_2(x, \mu)) \\ + \cos(\theta_2(x, \mu) + 2\theta_1(x, \mu)).$$

Gauge transformations must be applied iteratively to each site independently to maximize the global quantity. There are two local gauge conditions that must be met when the center gauge has

been obtained. The first condition comes from the  $\tilde{\lambda}_8$  contribution:

$$\sum_\mu \partial_\mu^- [\sin(2\theta_1(x, \mu) + \theta_2(x, \mu)) \\ + \sin(2\theta_2(x, \mu) + \theta_1(x, \mu))] = 0,$$

where  $\partial_\mu^-$  is the lattice derivative

$$\partial_\mu^- f(x) = f(x) - f(x - \hat{\mu}).$$

The second gauge condition to be met comes from the  $\lambda_3$  contribution:

$$\sum_\mu \partial_\mu^- [2 \sin(\theta_1(x, \mu) - \theta_2(x, \mu)) \\ + \sin(2\theta_1(x, \mu) + \theta_2(x, \mu)) \\ - \sin(2\theta_2(x, \mu) + \theta_1(x, \mu))] = 0.$$

Numerically, neither term can actually be zero. In practice, numerical iterations are continued until both terms are on the order of  $10^{-7}$ . In the continuum these two gauge conditions are equivalent to the Landau gauge:

$$\partial_\mu \theta_1(x, \mu) = 0$$

$$\partial_\mu \theta_2(x, \mu) = 0$$

### 3. Calculations

We generated 400 equilibrium  $SU(3)$  lattice configurations on a  $10^3 \times 16$  lattice with  $\beta = 5.9$ . At this value of  $\beta$  the correlation length is about 4 lattice spacings. Using these configurations and smearing techniques[6] we computed the heavy quark potential and measured the full  $SU(3)$  string tension,  $\sigma_f$ . We found  $\sigma_f = .068(3)$ .

Next, the 400 configurations were fixed to the maximal abelian gauge and the two species of abelian fields[7] were projected out. Overrelaxation was used with  $\omega = 1.7$ [8]. In each abelian configuration Dirac strings were located using the  $1 \times 1$  plaquettes and the two species of monopole configurations were obtained. The monopole configurations were used to compute the heavy quark potential (see Fig. (2) below). Results for both monopole species were found to be the same. The string tension from the potential computed using monopoles was found to be  $\sigma_{mon} = .047(2)$ . This is about 30% lower than the full  $SU(3)$  string

tension  $\sigma_f$ . We believe this is the first quantitative calculation of the  $SU(3)$  string tension from monopoles, although hints that the monopole string tension is low have appeared before[4].

After gathering the abelian configurations, we next applied our abelian center gauge fixing algorithm. No closed form solution of the parameters  $\alpha_1$  and  $\alpha_2$  could be found for the transcendental equations resulting from maximizing  $R_3$ . The  $\alpha$ 's were determined iteratively. The value of the normalized abelian trace computed from using the  $U(1) \times U(1)$  links was observed to change from an average value of 0.525 before center gauge fixing to 0.922 after gauge fixing.

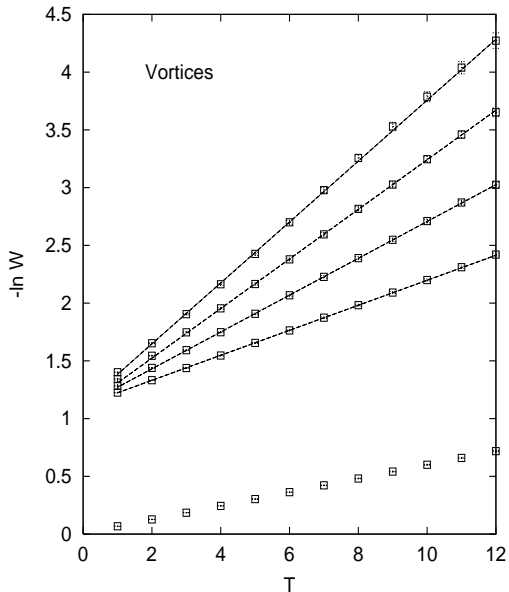


Figure 1. Wilson loops computed using  $Z(3)$  links obtained after indirect center projection. The loop values are for  $R = 1$  to  $R = 5$  ordered from bottom to top, respectively

From the center gauge fixed configurations, the links were replaced by the  $Z(3)$  phase each was closest to. Wilson loops were computed using the

$Z(3)$  configurations. The Wilson loop data is presented in Figure (1). Fits to the potential computed using the  $Z(3)$  Wilson loops gave a string tension of  $\sigma_{vort} = .060(4)$ . Thus, we find the string tension from indirect center projection to be too low.

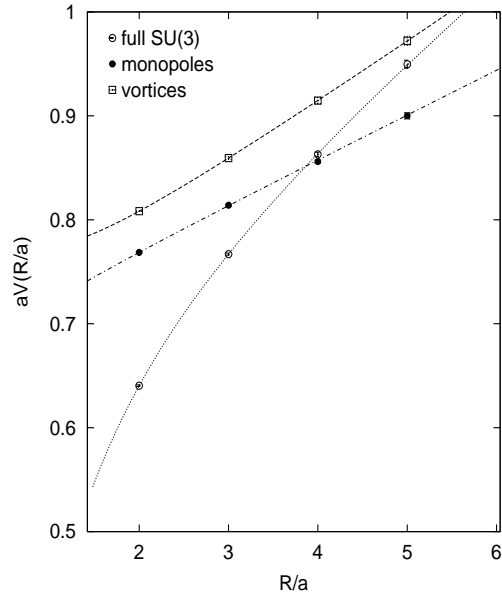


Figure 2. Comparison of the three heavy quark potentials, computed directly from the  $SU(3)$  gauge fields, using monopoles, and using projected center vortices.

In Figure (2) we compare the heavy quark potentials. The potentials computed from the monopoles and projected center vortices show little Coulomb behavior at small  $R$  as compared to the full  $SU(3)$  potential. This has been observed in other calculations in  $SU(2)$  and  $U(1)$ [3,5,10]. Calculations which demonstrate the confinement mechanism are not required to reproduce the short distance behavior of the potential, only the string tension.

#### 4. Conclusions

We have presented results for the string tension computed using monopoles and center vortices. The calculations use operations defined at the level of a single lattice spacing. Neither monopoles nor center vortices reproduce the full  $SU(3)$  string tension. There is still a need to refine the methods used here if one is to discard or accept monopoles or center vortices as the confinement mechanism in QCD. We conclude by discussing possible reasons to suspect systematic errors exist in these calculations.

Monopoles and vortices are computed using objects defined on the single link level. The results for the string tension may be corrupted by unphysical degrees of freedom due to misidentification of monopoles and center vortices by using operators defined at the single link level. Also, we may not be using volumes large enough to suppress finite volume effects since monopoles and center vortices are objects that likely extend over many lattice spacings. To investigate the first problem, we need to construct other methods for identifying monopoles and center vortices. For the second problem we need to work on larger lattices. Using larger lattices is difficult due to the computationally intensive nature of the gauge fixing methods.

The numerical procedures for the maximal abelian and center gauge fixing suffer from Gribov copy problems. In particular, for the direct and indirect center gauge fixing procedures in  $SU(2)$  the effects of Gribov copies on the numerical value of the string tension are large[11].

The effect of taking account of the Gribov ambiguity for  $SU(2)$  for all cases investigated so far, is to decrease the corresponding string tension. So for monopoles in the maximal abelian gauge, the monopole string tension for one gauge fix/configuration is quite close to the full  $SU(2)$  value [3], and decreases by  $O(10\%)$  when account is taken of the Gribov ambiguity by generating many copies/configuration [12]. Our result for the  $SU(3)$  monopole string tension is considerably lower than the full  $SU(3)$  value even at one gauge fix/configuration, and would presumably be reduced somewhat more when gauge ambigu-

ties are accounted for. Thus the  $SU(2)$  and  $SU(3)$  cases appear to be rather different. Given the large noise level of our data, we did not succeed in extracting a string tension at the  $U(1) \times U(1)$  level, before projecting to monopoles or vortices. Given our low results for the monopole and center vortex string tensions, this is now an important quantity to measure. A failure of abelian projection at the  $U(1) \times U(1)$  level for  $SU(3)$  would have important implications for ideas on confinement in  $SU(3)$ . We plan to return to this question in future work.

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